VECTOR ADDITION

PURPOSE:
To experimentally verify the rules for vector addition by graphical (scale drawing) and by components.

APPARATUS:
Force Table with Pulleys
50g mass holders
Assortment of masses from 1g to 100g

METHOD:
Forces produced by masses attached to strings over pulleys are vectors used in this experiment. The directions of the vectors can be varied by positioning the pulleys at different points around the rim of the circular force table. The magnitudes of the vectors are changed by varying the masses on the strings. At the center of the force table, all of the strings are attached to a plastic ring. (See Figure 1).

You are given two (or three) forces, which are called $F_a$ and $F_b$ (or $F_a$, $F_b$ and $F_c$), and your problem is to find their resultant $F_R$. $F_R$ is the result of the combined actions of $F_a$ and $F_b$ (or $F_a$, $F_b$ and $F_c$). What you actually find on the force table is the equilibrant, $F_E$ of the given forces: the force that balances them, so there is no tendency for the central ring to start moving when $F_E$ is applied along with the given forces. The resultant, $F_R$ has the same magnitude as $F_E$, and its direction is opposite to that of $F_E$. The direction of $F_R$ is found by subtracting 180° from the angle found for $F_E$.

PROCEDURE:
1. Given forces are $F_a = 500$ at 0° and $F_b = 500$ at 120°.
2. Place pulleys at the positions of the given forces and add the necessary masses to produce $F_a$ and $F_b$ (include the mass of the holder). Remove any unused mass holders from their strings.
3. Find the angle for $F_E$ by positioning another pulley roughly opposite the given forces and pulling the string with your hand over the pulley. If you cannot center the plastic ring on the central pin by pulling the string down over the pulley, the correct angle has not been found. Move the pulley for $F_E$ in steps of about 5° along the rim toward the direction where the ring touches the central pin, until you find the position where the ring can be centered by pulling the string over the pulley. It might be necessary to slide the strings slightly where they are tied to the ring to make the line of the string pass through the center of the pin. You may also need to use finer angular adjustments as you approach the proper position.
4. Put a mass holder on the string you have been pulling and add masses until the plastic ring is centered, with no tendency to move when you tap the force table with your hand.
5. Record all of the angles and masses with their uncertainties in accord with your teacher's instructions.

6. Repeat steps 2 through 5 for \( F_a = 300 \text{ at } 30^\circ \) and \( F_b = 400 \text{ at } 80^\circ \).

7. Repeat steps 2 through 5 for \( F_a = 200 \text{ at } 0^\circ \), \( F_b = 100 \text{ at } 70^\circ \) and \( F_c = 100 \text{ at } 160^\circ \).

8. Find the approximate error due to friction by putting 500g on one string and 500g on a second string directly opposite. Find how much mass must be added to one side for the ring to move noticeably
(a) When the mass is added gently.
(b) When the table is tapped as they are added.

**ANALYSIS:**

1. Find the experimental value of \( F_R \) for each of the three different setups. Remember that the magnitude of \( F_R \) is the same as that for \( F_E \) and that the angle for \( F_R \) is that of \( F_E \) minus 180°. Show any calculations and record these values on your calculations page.

2. Find \( F_R \) for each setup using the graphical method (scale drawing). Choose a reasonable and convenient length scale to represent the vector magnitudes, then using a protractor and ruler lay out the given vectors head to tail. \( F_R \) runs from the tail of the first to the head of the last vector. Use a scale sufficiently large that the completed drawing nearly fills half of an \( 8\frac{1}{2} \times 11 \) page. Label all magnitudes and angles on the drawing and list the scale used.

3. Find \( F_R \) by the method of components (using trigonometry) and show your work on the calculations page. See your text if you don't remember how to do this.

4. Find the percent difference comparing the magnitudes only of the experimental results to the component method results and also comparing the magnitudes of the graphical results to the component method results.

5. Calculate the percent error expected due to friction from your data in step 8-b.

Your results table should look something like this:

<table>
<thead>
<tr>
<th>Setup</th>
<th>( F_R ) Exp.</th>
<th>( F_R ) Graph.</th>
<th>( F_R ) Comp.</th>
<th>% Diff. Experimental</th>
<th>% Diff. Graphical</th>
<th>Friction % error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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</tbody>
</table>

**Vector Addition**
**Graphical Method**

An example of the use of the head-to-tail method is illustrated below. The problem involves the addition of three vectors:

20 m, 45 deg. + 25 m, 300 deg. + 15 m, 210 deg.

![Graphical Method Diagram]

**SCALE: 1 cm = 5 m**

The head-to-tail method is employed as described above and the resultant is determined (drawn in red). Its magnitude and direction is labeled on the diagram.

**SCALE: 1 cm = 5 m**

![Component Method Diagram]

**Component Method**
Now we will add the same three vectors above mathematically. Once again, they are: \( \mathbf{A} = 20\text{m}@45\text{deg.}, \mathbf{B} = 25\text{m}@300\text{deg.}, \) and \( \mathbf{C} = 15\text{m}@210\text{deg.} \)

First draw a sketch of the vectors:

Then find the \( x \) components:

\[
\begin{align*}
\mathbf{A}_x &= \mathbf{A}\cdot\cos\theta_A = 20\text{m}\cdot\cos(45\text{deg}) = 14.1\text{m} \\
\mathbf{B}_x &= \mathbf{B}\cdot\cos\theta_B = 25\text{m}\cdot\cos(300\text{deg}) = 12.5\text{m} \\
\mathbf{C}_x &= \mathbf{C}\cdot\cos\theta_C = 15\text{m}\cdot\cos(210\text{deg}) = -13.0\text{m}
\end{align*}
\]

\[ \mathbf{R}_x = \mathbf{A}_x + \mathbf{B}_x + \mathbf{C}_x = 13.6\text{m} \]

Find the \( y \) components:

\[
\begin{align*}
\mathbf{A}_y &= \mathbf{A}\cdot\sin\theta_A = 20\text{m}\cdot\sin(45\text{deg}) = 14.1\text{m} \\
\mathbf{B}_y &= \mathbf{B}\cdot\sin\theta_B = 25\text{m}\cdot\sin(300\text{deg}) = -21.7\text{m} \\
\mathbf{C}_y &= \mathbf{C}\cdot\sin\theta_C = 15\text{m}\cdot\sin(210\text{deg}) = -7.5\text{m}
\end{align*}
\]

\[ \mathbf{R}_y = \mathbf{A}_y + \mathbf{B}_y + \mathbf{C}_y = -15.1\text{m} \]

Now that you have the components of the resultant vector, use the Pythagorean theorem to find the magnitude of the resultant and the inverse tangent function to find the direction angle:

\[ \mathbf{R} = \sqrt{\mathbf{R}_x^2 + \mathbf{R}_y^2} = \sqrt{(13.6\text{m})^2 + (-15.1\text{m})^2} = 20.2\text{m} \]

\[ \theta_R = \tan^{-1}\left( \frac{\mathbf{R}_y}{\mathbf{R}_x} \right) = \tan^{-1}\left( \frac{-15.1\text{m}}{13.6\text{m}} \right) = -48^\circ \]

\(-48^\circ\) is the same as a positive \( 312^\circ \). So our resultant vector is \( \mathbf{R} = 20.2\text{m}@312^\circ \).

This shows that the graphical method used above was fairly accurate but not exact.