Measurement of Capacitance

Pre-Lab Questions Page

1. A capacitor is used to store ________________.

2. What is the SI unit for capacitance?

3. A capacitor basically consists of two metal plates separated by an insulating material called a ________________.

4. The reactance of the capacitor depends on ________________.

5. The dielectric constant of a material is symbolically represented by _______.

6. Calculate the equivalent capacitance of an 8.0 pF and 12 pF capacitor connected in (a) series and (b) parallel.

7. Many electronic components carry warning labels that say, “DANGER: Hazardous voltage inside. Do not open.” What is there inside these devices that makes them so dangerous? Why wouldn’t you be safe if you unplugged the equipment before opening the case?
MEASUREMENT OF CAPACITANCE
Bridge Method

OBJECTIVE: To study the bridge method of measuring capacitance and to check formulas for the capacitance of capacitors connected in series and in parallel.

APPARATUS:
1. Standard capacitor \( C_1 \) in the range 0.1-1.0 \( \mu \text{F} \)
2. Two unknown capacitors \( C_2 \) in the above range
3. Decade resistance box (1000\( \Omega \))
4. Decade resistance box (10000\( \Omega \))
5. Audio signal generator
6. Telephone receiver or high-impedance headphones

INTRODUCTION:
A capacitor basically consists of two metal plates separated by an insulating material called a dielectric. Such an arrangement stores electric charge by reason of the fact that, if a voltage source is connected between the plates, positive charge flows onto one plate from the other, leaving the latter negative. This process continues until the field in the dielectric is strong enough to stop the further flow of charging current. At this point, a certain amount of charge (positive in one case and negative in the other) is stored on each plate, and a potential difference equal to the source voltage exists between them. The ratio between this charge and this potential difference is a constant for a given capacitor and is called capacitance. Thus,

\[
C = \frac{Q}{V} \quad \text{eq. 1}
\]

where \( C \) is the capacitance in farads, \( Q \) is the charge in coulombs, and \( V \) is the potential difference in volts.

When a capacitor is connected in an ac circuit, the alternate charging and discharging make it appear to pass an ac current. A linear relationship between the ac voltage applied and the apparent current passed that is very similar to Ohm’s law for resistance is found to hold. The proportionality factor, \( X_c \), between this voltage and current is called the \textit{capacitive reactance} of the capacitor and is measured in ohms, \( \Omega \), as is resistance. However, it is related to the capacitance by

\[
X_c = \frac{1}{2fC\pi} \quad \text{eq. 2}
\]

Where \( X_c \) is the reactance of the capacitor in ohms, \( C \) is the capacitance in farads as before and \( f \) is the frequency in hertz of the applied ac. Thus the reactance depends on frequency and becomes infinite as frequency goes to zero, an expression of the fact that for dc (\( f = 0 \)) a capacitor is in fact an open circuit. However, for ac of a particular frequency, a capacitor behaves in many ways like a resistor.
The reactance \( X_{c1} = 1/(2\pi f C_1) \) and \( X_{c2} = 1/(2\pi f C_2) \) and the resistances from figure 1 have the following relationship

\[
\frac{C_2}{C_1} = \frac{R_3}{R_4}
\]

This relation holds when the balance condition, that is, a null indication of the ac detector. With \( R_3 \) and \( R_4 \) known, \( C_2 \) can be found in terms of \( C_1 \), a standard capacitor whose value is known to reasonably high precision. Notice the inverse relationship between the capacitances and the resistances, due to the inverse proportionality of capacitance and its associated reactance.

This inverse proportionality can be used to obtain the formulas for the capacitance of a series or parallel arrangement of capacitors. Suppose three capacitors are connected in series as shown in Fig. 2. In this ac circuit, they exhibit reactances \( X_{c1}, X_{c2}, \) and \( X_{c3} \), and the total reactance of the combination is the sum of these individual reactances. Thus,

\[
X_c = X_{c1} + X_{c2} + X_{c3}
\]

Substitution of the expression for reactance, Equation 2, and multiplying through by \( 2\pi f \) leads to the conclusion that the capacitance \( C \) exhibited by the three capacitors in series is given by

\[
C = \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right)^{-1}
\]

Figure 1. Bridge method of comparing capacitors

Similarly, with capacitors in parallel as in Fig 3, we should find that
\[
\frac{1}{X_C} = \frac{1}{X_{C1}} + \frac{1}{X_{C2}} + \frac{1}{X_{C3}}
\]

that there is an equivalent capacitance

\[ C = C_1 + C_2 + C_3 \quad \text{Eq. 5 parallel connection} \]

Figure 3 Capacitors in parallel

The capacitance bridge of Fig 1 can also be used to measure the so-called dielectric constant of an insulating material. The capacitance of a parallel plate capacitor is given by the expression

\[ C = \varepsilon_0 \kappa A/d \quad \text{eq. 6} \]

where \( C \) is the capacity in farads (multiply by 1 million or \( 10^6 \) for microfarads), \( A \) is the area of either plate in square meters, \( d \) is the plate separation in meters, \( \varepsilon_0 \) is the permittivity of free space and \( \kappa \) is the dielectric constant of the material between the plates. Note that \( C \) varies directly with \( \kappa \). If in our bridge we use two parallel-plate capacitors of identical dimensions, one of which \((C_1)\) has air (\( \kappa \) very nearly equal to 1) or, even better, a vacuum (\( \kappa \equiv 1 \)) between the plates while the other \((C_2)\) has the space between the plates filled with the insulator under test, \( \kappa \) for that material will be equal to \( C_2/C_1 \) and will be given directly by the ratio \( R_3/R_4 \) at balance.

In this experiment, a standard capacitor will be used as \( C_1 \) and an unknown capacitance inserted at \( C_2 \). Resistors \( R_3 \) and \( R_4 \) will be decade resistance boxes, which will be adjusted to obtain the balance condition. The unknown capacitance can then be calculated from the known value of \( C_1 \) and the values of \( R_3 \) and \( R_4 \) read from the boxes. Two unknown capacitors will be measured in this manner, after which series and parallel combinations of these capacitors will be measured to check the validity of Equations 4 and 5.

**PROCEDURE:** Use the data table to record your answers

1. Begin by connecting the telephone receiver directly to the signal generator output terminals. Turn the signal generator on, set the output level for comfortable listening, and tune the generator frequency to optimum value. This will be the setting for which the sound heard in the receiver is loudest. Telephone receivers have a limited frequency range and are made to be most sensitive in the range occupied by the human voice. What you have just done is to find the frequency for which your receiver is most sensitive and will thus perform best as a null detector. Remember that you are going to adjust \( R_3 \) and \( R_4 \) to give no audible sound in the receiver, and the
more sensitive the receiver is the more precise that adjustment can be. For ordinary telephone receivers and communication (not high-fidelity) headsets, the optimum frequency is around 800 – 1000 Hz.

2. Wire your capacitance bridge as shown in Fig 1. Use the 10,000-$\Omega$ decade box for $R_3$ and set it to 200-$\Omega$. Use the 1000-$\Omega$ decade box for $R_4$, the standard capacitor for $C_1$, and one of the unknown capacitors for $C_2$.

3. Set the signal generator output at maximum, check that it is still tuned to the optimum frequency found in Procedure 1, and listen for the tone in the telephone receiver.

4. Leave $R_3$ set at 200-$\Omega$ and adjust $R_4$ for minimum audible signal. Record the value obtained along with the value for $R_3$ and that of the standard capacitor ($C_1$).

5. Repeat Procedure 2 – 4 with the other unknown capacitor substituted at $C_2$. Record $R_4$ for this case.

6. Repeat Procedure 2 – 4 with the two unknown capacitors in series at $C_2$. Record $R_4$ for this case.

7. Repeat Procedure 2 – 4 with the two unknown capacitors in parallel at $C_2$. Record $R_4$ for this case.

**CALCULATIONS:** Use the data table to record your answers

1. Compute the capacitance of each of the unknown capacitors for data of Procedures 2 – 5, respectively.

2. From the data of Procedure 6, find the capacitance of the series combination of the two unknown capacitors.

3. Use Equation 4 and your measured values of the unknown capacitors from Calculation 1 to obtain a calculated value for the capacitance of the series combination. Compare this value with that obtained in Calculation 2 by finding the percent error. Recall: $\text{%error} = \left| \frac{A - E}{A} \right| \times 100$

4. From data of Procedure 6, find the capacitance of the parallel combination of the unknown capacitors.

5. Use Equation 5 and your measured capacitance values to obtain a calculated value for the capacitance of the parallel combination. Compare this value with that obtained in Calculation 4 by the percent difference.
DATA:

Value of $R_3$ __________________  Value of standard capacitor ($C_1$) __________

<table>
<thead>
<tr>
<th>Capacitors measured</th>
<th>$R_4$</th>
<th>Experimental capacitance ($C_2$)</th>
<th>Known capacitance</th>
<th>Percent error %error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unknown no. 1</td>
<td></td>
<td>0.1 µF</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unknown no. 2</td>
<td></td>
<td>0.4 – 0.5 µF</td>
<td></td>
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<td>Series combination</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Parallel combination</td>
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</tbody>
</table>

QUESTIONS:

1. Do the results you obtained experimentally for the series and parallel connections of capacitors check with the laws for such connections within experimental error of 10%?

2. What would be the effect on the capacitance of a parallel-plate capacitor if a sheet of pyrex glass were introduced between the plates? Assume the dielectric to be air (dielectric constant $\kappa \approx 1$) before the glass is inserted.

3. Assuming that the charge remains the same, what would be the effect on the potential difference between the plates of a parallel-plate capacitor if a sheet of pyrex glass were introduced between the plates?

4. Which factor in Equation 6 is varied in the tuning capacitors used in radio sets?

5. Three capacitors with capacitances of 2, 3, and 5 µF, respectively, are joined in parallel. The combination is connected in series with a 5 µF capacitor. What is the equivalent capacitance of the final arrangement?

6. Why did you use the maximum output of the signal generator to power your bridge?

7. A parallel plate capacitor is made up of two circular plates separated by a sheet of mica. The plates have a diameter of 20 cm, and the mica is 0.1 mm thick and has a dielectric constant of 3. Calculate the capacitance in units of microfarads.