Mathematics Educators of Greater St. Louis
and St. Louis Community College
at Florissant Valley present

Excellence In Mathematics

Seventh Grade Test
Thirty-Eighth Annual Mathematics Contest
March 19, 2016

I. Do not open the test booklet or begin work until instructed to do so by your proctor.

II. You have 75 minutes to take this test.

III. Listen carefully as the proctor explains where to write your name, the name of your school, your grade level, and how to mark your answers.

IV. You may use a calculator. You only need a four-function calculator, but you may use any calculator approved for the SAT test, which includes most graphing calculators except the TI-92 and TI-Voyager. If you are unsure whether your calculator is allowed, check with your proctor.

V. Your score will be the number of questions you answer correctly. In the event of ties, Problem #40 will be used as a tie-breaker. If ties still remain, Problem #39 will be used as a tie-breaker and so on until all ties are broken.
1. The perimeter of a rectangle is 36 cm. If the length of one side of the rectangle is 12 cm, what is the area of the rectangle?
   A. 30 cm²  B. 36 cm²  C. 48 cm²  D. 72 cm²  E. 144 cm²

2. Which one of the following five numbers can be written as the sum of three distinct non-zero square numbers?
   A. 34  B. 38  C. 44  D. 47  E. 55

3. With $5.25 in nickels and quarters, you have twice as many nickels as quarters. What is the number of coins that you have?
   A. 15  B. 30  C. 36  D. 45  E. 48

4. Determine the smallest positive difference between any pair of these numbers.
   \[ 0.9 \quad 0.79 \quad 1.12 \quad 1.043 \quad 0.9714 \]
   A. 0.11  B. 0.071  C. 0.077  D. 0.0714  E. 0.0716

5. On January 13, 2016, the NFL gave Stan Kroenke permission to move the St. Louis Rams to Inglewood, California, about 1860 miles away. Starting at 8 AM on January 13, St. Louis Rams fans ran Stan Kroenke out of town at an average rate of 9 miles per hour, 12 hours a day – from 8 AM to 8 PM. Using this data, on what day did Stan Kroenke arrive in Inglewood?

6. Consider a pizza to be a cylinder of height \( a \) and radius \( z \). What is the volume of this pizza?
   A. \( 2\pi z a \)  B. \( \pi a^2 z \)  C. \( 4\pi z^2 a \)
   D. \( \pi z a z z \)  E. \( \pi z z a \)

7. What is the sum of all of the numbers in both of these finite arithmetic sequences?
   \[ 2; 12; 22;….; 92; 102 \quad \text{and} \quad 8; 18; 28;….; 98; 108 \]
   A. 1100  B. 1120  C. 1210  D. 1220  E. 1320

8. In a 48-minute basketball game, the Lakers had the ball 5/8 of the time. When the Lakers had the ball, their point-guard Kobe had the ball 1/3 of the time. For what per cent of the game did Kobe have the ball? Round to the nearest per cent.
   A. 20%  B. 21%  C. 22%  D. 23%  E. 25%

9. The Kansas City Federal Reserve Money Museum Bank has a display of 40 million dollars in stacks of $100 bills. It costs the US Mint 6.2 cents to make one bill of any US denomination. How much did it cost to make the bills in this display of 40 million dollars?
   A. $248  B. $2,480  C. $24,800  D. $248,000  E. $2,480,000
10. A parking meter “runs out of time” when its needle arrives at 0. Assume that the marks on the meter are equally spaced. In approximately how many minutes will this parking meter “run out of time”?

A. 9  B. 18  C. 30  D. 36  E. 84

11. The prime factorization of 2016 is \(2016 = 2^L \times 3^M \times 7^N\). What is the sum \(L + M + N\)?

A. 6  B. 7  C. 8  D. 9  E. 10

12. Ben receives a 5% discount on a $60 item and a 15% discount on a $120 item. Overall, what per cent discount did Ben receive? Round to nearest tenth of a per cent.

A. 10.0%  B. 11.7%  C. 12.2%  D. 12.5%  E. 20%

13. For whole numbers \(A\) and \(B\), \(3^A + 5^B = 74,674\). What is the sum \(A + B\)?

A. 12  B. 13  C. 14  D. 15  E. 16

14. ABC is an equilateral triangle of side-length 8 cm. An irregular hexagon is formed by cutting equilateral triangles of side-length 1 cm from vertex A, of side-length 2 cm from vertex B, and of side-length 3 cm from vertex C. What is the perimeter of the hexagon?

A. 12 cm  B. 15 cm  C. 18 cm  D. 21 cm  E. 24 cm

15. Horse races are measured in furlongs. A furlong is 1/8 of a mile. How much longer is a 1.5 mile horse race than a 7 furlong race?

A. \(\frac{5}{8}\) mile  B. \(\frac{1}{8}\) mile  C. \(\frac{3}{8}\) mile  D. \(\frac{1}{2}\) mile  E. \(\frac{1}{4}\) mile

16. This number line from \(-10\) to 20 has been divided into equal parts. What is the sum \(A + B\)?

A. 8  B. 10  C. 11  D. 12  E. 24

17. One of the unplanned effects of China’s one-child policy was that in 2015 there were 33 million more males than females in China. With a 2015 population of China of 1.375 billion, what per cent of Chinese were male? Round to the nearest tenth of a per cent.

A. 50.8%  B. 51.2%  C. 51.6%  D. 51.8%  E. 52.4%

18. For the given triangle ABC, select the best estimate of the sum of the measures of angles B and C.

A. 45°  B. 75°  C. 90°  D. 100°  E. 120°
19. When 2016 is written as the product of two whole numbers, what is the smallest possible positive difference between those two numbers?

A. 6  B. 12  C. 20  D. 29  E. 44

20. Six circles fit snugly in a rectangle with length 72 cm. What is the total area of all of the shaded regions? Round to the nearest square centimeter.

A. 742 cm$^2$  B. 1208 cm$^2$  C. 2470 cm$^2$
D. 3004 cm$^2$  E. 7401 cm$^2$

21. A waiter serves Fran black coffee in a cup which is a cylinder of diameter 7.6 cm and height 9 cm. This cup is 90% full of coffee but Fran intends to add a LOT of half-and-half and asks for a larger cup. The waiter pours all of Fran’s coffee into a larger cup which is a cylinder of diameter 8.8 cm and height 10.4 cm. Before Fran adds half-and-half, what per cent full is the second cup? Round to the nearest per cent.

A. 58%  B. 65%  C. 67%  D. 74%  E. 78%

22. If $x = -3$, what is the sum of these four expressions?

$x^2$; $x^3$; $-2x$; $-5 - x$

A. −14  B. −20  C. −26  D. −32  E. 4

23. On this square grid, how many of the five triangles are isosceles triangles?

Assume that all vertices of the triangles lie on lattice points of the graph.

A. 1  B. 2  C. 3
D. 4  E. 5

24. For how many whole numbers $N$ is the number $\frac{N}{2016}$ between 0.7 and 0.8?

A. 200  B. 201  C. 202  D. 203  E. 204
25. Using any number of non-overlapping 1x1 and 1x2 tiles, in how many different ways can a 1x6 rectangle be covered? Note: Solutions with all 1x1 or all 1x2 tiles are allowed.

- A. 8
- B. 9
- C. 11
- D. 12
- E. 13

26. While visiting grandma and grandpa for Christmas in the Upper Peninsula, grandma would call her son, daughter-in-law, and granddaughter for dinner by calling the three of them in any sequence. She called: her son Rick, Rich, or Richard; her daughter-in-law Fran or Francine; and her granddaughter Suzanne or Zan.

In how many different ways could grandma call the three of them to dinner?

- A. 12
- B. 24
- C. 36
- D. 54
- E. 72

27. A three-dimensional shape is formed by placing a pyramid ABCDP on top of a cube on face ABCD. What is the total number of faces, edges, and vertices of this shape?

- A. 30
- B. 34
- C. 35
- D. 36
- E. 44

28. Rick’s family trip in April 2015 to Sarawak, Malaysia, was his first return since 1974. On this trip, the government mandated gasoline price was 1.95 Malaysian Ringgit per liter. If one US dollar equals 4.425 Ringgit and 1 liter equals 1.06 quarts, what was the price of gasoline in dollars per gallon?

Round to the nearest cent.

- A. $0.42
- B. $1.66
- C. $1.87
- D. $2.29
- E. $8.14

29. As shown, squares are mounted on the three sides of an isosceles right triangle ADG. The area of square ABCD is 15 square centimeters. What is the area of the polygon ABCDEFGHJ?

Round to the nearest tenth of a square centimeter.

- A. 52.5 cm²
- B. 58.7 cm²
- C. 62.3 cm²
- D. 67.5 cm²
- E. 75.0 cm²

30. Earl will attach narrow (grey) slats for the lower shelf of a table. As shown in his design, the interior length between the boards that form the frame is 104 3/8 inches. Each slat is 1 3/4 inches wide and there is a gap of 3/8 inches between slats. The distance between the last slat and the frame at each end will be 5/16 inch.

To allow for mistakes, Earl will order at least 10% more slats than the minimum number needed for the table. Including the slats shown in the diagram, what is the least number of slats that Earl must order?

- A. 48
- B. 49
- C. 53
- D. 54
- E. 66
31. The distance from the center of a clock to the tip of its minute hand is 8 cm. From 8:45 AM to 6:30 PM of the same day, how many centimeters does the tip of the minute hand move? Round to the nearest centimeter.

A. 40 cm  B. 41 cm  C. 475 cm  D. 480 cm  E. 490 cm

32. Three mice are at the three vertices of an equilateral triangle. At the same time, each mouse runs to an adjacent vertex. To determine which vertex to run to, each mouse flips a fair coin:
- If Heads, that mouse runs clockwise
- If Tails, that mouse runs counter-clockwise.

After they do this once, what is the probability that there are NOT two mice at any one vertex?

A. 1/8  B. 1/4  C. 1/2  D. 3/4  E. 1

33. The diagram depicts a shoe-lace pattern for a shoe with four pairs of eyelets. \(x\) and \(y\) are the horizontal and vertical distances between eyelets, as shown. The same amount \(L\) of lace is left at each end for tying.

Austin uses the same pattern for his shoes with six pairs of eyelets. When the lace is tightened, \(x = 3\) cm and \(y = 2\) cm. If Austin buys a shoelace of length 60 cm, calculate \(L\), the length of each end left for tying. Round to the nearest tenth of a centimeter.

A. 6.9 cm  B. 10.5 cm  C. 13.1 cm  D. 16.5 cm  E. 18.5 cm

34. \[4^{2016} + 8^{2016} = 2^M \left(1 + 2^N\right)\] What is the sum \(M + N\)?

A. 2016  B. 4032  C. 6048  D. 8064  E. 12,096

35. Among the 400 athletes in a college, every 3 months 100 of them are randomly selected for testing for illegal drugs. Therefore, in one year, any one athlete could be tested 0, 1, 2, 3, or 4 times. In one year, what is the probability that their track star Breanna is tested at least once? Round to the nearest per cent.

A. 59%  B. 68%  C. 75%  D. 84%  E. 100%

36. \(B\) and \(C\) are digits, possibly the same digit, in the 7-digit number: 1,68B,53C.

For how many ordered pairs \((B, C)\) is 1,68B,53C a multiple of 15?

A. 3  B. 4  C. 5  D. 6  E. 7
37. In this multiplication problem, each letter represents a digit 0 through 9. Neither A nor E represent 0. Two different letters may represent the same digit. What does the sum A + B + C + D + E equal?

A. 22 B. 23 C. 25 D. 27 E. 29

38. For the 9 students taking the AIME exam, their math teacher seats them as shown so that no two students are sitting in the same row, column, or diagonal. When the teacher briefly leaves the room, exactly 3 students move to seats adjacent (horizontally, vertically, or diagonally) to their assigned seats. Since again no two students are sitting in the same row, column, or diagonal, the teacher doesn’t notice that anyone has moved.

If D was one of the students who moved, who were the other two students who moved?

A. A and C B. C and G C. E and G D. E and I E. F and G

39. Supergirl is trapped motionless in an equilateral triangle as it revolves clockwise around a regular octagon of the same side length. The first “step” is shown. As the triangle pivots at point B: Vertex C rotates to C’ and vertex A rotates to A’. After 8 “steps”, the triangle is again on top of the octagon.

As the triangle revolves, how many 360° rotations has Supergirl experienced?

A. $\frac{2}{3}$ B. $\frac{2}{3}$ C. $\frac{3}{3}$
D. $\frac{3}{3}$ E. 4

*Problem #40 is on the next page.*
40. A completed 5x5 Numbrix Puzzle is shown to the right.

Notice that the numbers 1 through 25 are positioned so that it is possible to draw a continuous path from 1 to 25.

All segments of the path must connect squares horizontally or vertically, not diagonally.

Complete the Numbrix Puzzle below so that the 49 cells contain the numbers 1 through 49 and so that it is possible to draw one continuous path from 1 to 49 which connect squares horizontally or vertically, not diagonally.

When finished, what is the sum of the numbers in the four cells marked A, B, C, and D?

A. 104  B. 108  C. 112  D. 114  E. 116