

3 x 13

*Mathematics Educators of Greater St. Louis
and St. Louis Community College at Florissant Valley present*

Excellence In Mathematics

Thirty-Ninth Annual Mathematics Contest

Eighth Grade Test ----- March 18, 2017

- I. Do not open the test booklet or begin work until instructed to do so by your proctor.
- II. You have 75 minutes to take this test. There are 36 problems.
- III. Listen carefully as the proctor explains where to write your name, the name of your school, your grade level, and how to mark your answers.
- IV. You may use a calculator. You only need a four-function calculator, but you may use any calculator approved for the SAT test, which includes most graphing calculators except the TI-92 and TI-Voyager. If you are unsure whether your calculator is allowed, check with your proctor.
- V. Phones or other electronic devices may NOT be used for any purpose.
- VI. Your score will be the number of questions you answer correctly. In the event of ties, Problem #36 will be used as a tie-breaker. If ties remain, Problem #35 will be used as a tie-breaker and so on until all ties are broken.

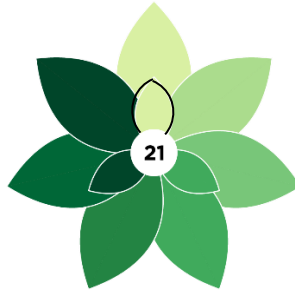
Note: More *Botanical Prime* art works of *Nicholas Rougeux* are at: <https://www.c82.net/work/?id=352>

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- Of all positive factors of 24, one is chosen randomly. What is the probability that it is odd?
A. $\frac{2}{7}$ B. $\frac{1}{8}$ C. $\frac{1}{4}$ D. $\frac{1}{6}$ E. $\frac{1}{7}$
- How many integers lie between $2 - \sqrt{50}$ and 3π ?
A. 12 B. 13 C. 14 D. 15 E. 16
- On a poster, Nick Rougeux has created a beautiful way to depict the prime factorization of a number. His depictions for 7, 21, and 24 are shown.



7
7 petals



3 x 7
10 petals



2 x 2 x 2 x 3
9 petals

Following the same pattern, how many petals would Nick's flower for "36" have?

- A. 10 B. 11 C. 12 D. 13 E. 15
- How many ounces are there in P pounds and Q ounces?
A. $16P + Q$ B. $\frac{P}{16} + Q$ C. $P + Q$ D. $P + \frac{Q}{16}$ E. $P + 16Q$
- Of an \$8,000 prize, Zeb received 40% while Xena received \$300 more than Caitlyn. How much more money did Zeb receive than Caitlyn?
A. \$500 B. \$650 C. \$800 D. \$950 E. \$1050
- Determine the positive difference between the *reciprocal of the sum of* $4\frac{1}{3}$ *and* $2\frac{1}{6}$ *and the sum of the reciprocals of* $4\frac{1}{3}$ *and* $2\frac{1}{6}$.
A. $\frac{151}{26}$ B. $\frac{7}{13}$ C. $\frac{9}{13}$ D. $\frac{13}{7}$ E. $\frac{13}{9}$
- X is a real number. The first three terms of an arithmetic sequence are given. What is the sixth term?
X + 5; 2X + 18; 4X + 20;
A. 128 B. 130 C. 132 D. 134 E. 136

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8. A 2-pound bag of oranges costs \$3.24 and a 3-pound bag of apples costs \$3.80. You buy a total of 36 pounds of oranges and apples. If the weight of the apples you buy is twice the weight of the oranges you buy, what is the total cost for your 36 pounds of apples and oranges?

A. \$48.72 B. \$49.84 C. \$51.96 D. \$52.86 E. \$53.64

9. For how many whole numbers N are $\frac{N}{50} > \frac{1}{2}$ and $\frac{N}{60} < \frac{2}{3}$?

A. 13 B. 14 C. 15 D. 16 E. 17

10.

Age (years)	11	12	13	14	15
Number of Students	8	6	7	1	3

What is the positive difference between the mean age and the median age of this group of students?

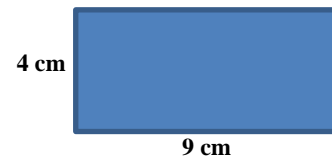
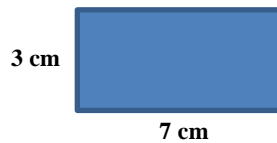
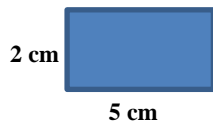
A. 0 B. 0.4 C. 0.6 D. 1.0 E. 1.4

11. M and N are positive integers such that the product $MN = 4000$. Neither number, M nor N , contains the digit 0. What is the sum of the digits of both M and N ?

A. 12 B. 13 C. 15 D. 16 E. 17

12. The first three rectangles in a sequence are shown. After the first rectangle, the next rectangle in the sequence is 2 cm longer and 1 cm wider than the previous rectangle.

In square centimeters, what is the area of the 20th rectangle in this sequence?



A. 504 cm² B. 860 cm² C. 861 cm² D. 903 cm² E. 990 cm²

13. If $6x - 3y = 4$ and $5x - 2y = -7$, what does $x - y$ equal?

A. 11 B. -2 C. $71/3$ D. $-23/3$ E. $-41/3$

14. Liz has averaged 94% on her homework. Her first four test grades were: 78%; 71%; 73%; and 76%. To compute each student's final grade, Liz's teacher uses the formula: $0.3H + 0.7T$, where H represents her homework average (in %) and T represents her test average (in %). Liz has two tests left in the term and no more homework.

What must Liz average on her last two tests to earn a final grade of 80%? Round to the nearest per cent.

A. 72% B. 73% C. 78% D. 81% E. 83%

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15. After their California wedding in June, Zan and Jonas took a 4-month trip. The lengths of their flights are recorded in the table below.

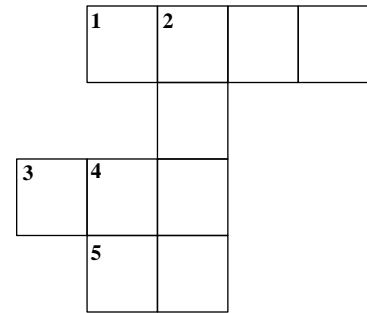
<u>Leg</u>	San Francisco	Anchorage	New York City	Windhoek Namibia	Johannesburg South Africa	Cairo Egypt	Santiago Chile	Saint Louis
	Anchorage	New York City	Windhoek Namibia	Johannesburg South Africa	Cairo Egypt	Santiago Chile	Saint Louis	San Francisco
<u>Distance</u> (km)	3240	5430	11700	1180	6230	12800	8230	2810

The Earth is approximately a sphere with a radius of 6370 km. The total flight distance of their trip is equal to how many times around the Earth at the equator? Round to the nearest tenth.

- A. 0.8 B. 1.1 C. 1.3 D. 1.5 E. 1.8
16. What is the sum of this series: $1 + 2 - 3 + 4 + 5 - 6 + 7 + 8 - 9 + \dots + 58 + 59 - 60$?
- A. 510 B. 525 C. 540 D. 570 E. 600
17. In a basketball game, Steph, Kevin, Klay, and Draymond scored all their team's points. Draymond scored $\frac{1}{6}$ of their points, Steph scored $\frac{2}{5}$ of their points, Kevin scored twice as many points as Draymond, and Klay made four 3-point baskets and no other points. How many points did Draymond score?
- A. 12 B. 15 C. 18 D. 20 E. 25
18. A jar is $\frac{4}{5}$ full of R red and Y yellow jelly beans in the ratio of R:Y = 5:3 . Blue jelly beans are added to fill the jar. What percent of the beans in the full jar are yellow?
- A. 25% B. 30% C. $33\frac{1}{3}\%$ D. 35% E. 37.5%
19. For trapezoid ABCD, angles A and B are right angles. AB = 9 cm and CD = 15 cm. If the area of the trapezoid is 99 square centimeters, what is its perimeter?
- A. 29 cm B. 34 cm C. 41 cm D. 46 cm E. 54 cm
20. Becca rents a car for D days and D is greater than 7. The fees are M dollars per day for the first 7 days and half that rate for each additional day. Find the total charge for Becca's rental for D days.
- A. $\frac{7}{2}M + \frac{1}{2}MD$ B. $\frac{3}{2}MD - \frac{7}{2}M$ C. $\frac{3}{2}MD - 7M$
- D. $7M + \frac{1}{2}MD$ E. $\frac{3}{2}MD$
21. Using three different numbers from this set, $\{-10, -6, -2, 4, 8\}$, what is the least possible value of $\frac{A-B}{C}$?
- A. -9 B. -8 C. $-4\frac{1}{2}$ D. $-1\frac{2}{5}$ E. $\frac{3}{5}$

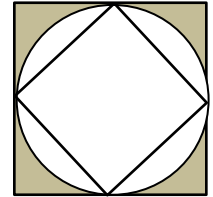
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22. In this *Cross-Number Puzzle*, the clue for each of the five numbers [1-Across; 3-Across; 5-Across; 2-Down; and 4-Down] is “**Power of 2**”. What is the sum of all ten digits in the finished puzzle?



- A. 38 B. 39 C. 40
D. 41 E. 42

23. A square is inscribed in a circle which is inscribed in a square. The sum of the areas of the four shaded regions is 10 square centimeters. What is the area of the smaller square? Round to the nearest square centimeter.



- A. 19 cm² B. 20 cm² C. 21 cm² D. 22 cm² E. 23 cm²

24. From the Norman Conquest of 1066 until the year 1971, the British money system included Pounds (£), Shillings (s), and Pence (d). *12 Pence equaled 1 shilling and 20 Shillings equaled 1 Pound.*

In 1965, a bookseller was given **20 £** for the purchase of three books each worth **4 £, 8 s, 6 d**. How much change did the customer receive?

- A. 6 £, 5 s, 4 d B. 6 £, 14 s, 6 d C. 6 £, 15 s, 4 d D. 6 £, 14 s, 4 d E. 7 £, 5 s, 6 d

25. The Fibonacci Sequence: *1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 134, ...* provides surprisingly accurate conversions between *miles per hour* and *kilometers per hour*. For example, 8 mph \approx 13 kph and 55 mph \approx 89 kph.

Using the conversion: *1 mph \approx 1.6093 kph*, what is the percent error when 55 kph is used for an estimate of 34 mph? Round to the nearest tenth of a percent.

- A. 0.1% B. 0.5% C. 1.2% D. 1.5% E. 5.2%

26. In triangle ABC, points D, E, and F lie on sides AB, BC, and CA respectively such that BD = BE and CE = CF. If the measure of angle A is 74°, then the measure of angle DEF is

- A. 50° B. 53° C. 56° D. 58° E. Cannot be determined

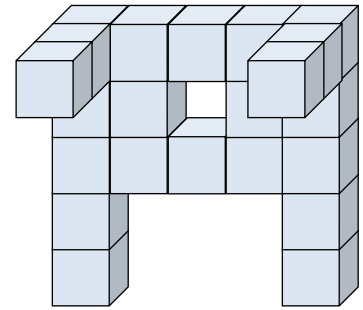
27. **x**, **y**, and **z** are non-zero integers.

How many different values are possible for this expression $\frac{x}{|x|} + \frac{2y}{|y|} + \frac{z}{|z|} + \frac{xyz}{|xyz|}$?

- A. 3 B. 4 C. 5 D. 6 E. More than 6

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28. This “creature” consists of the 22 one-centimeter cubes that you can see. What is the surface area of the creature?



- A. 68 cm^2 B. 72 cm^2 C. 76 cm^2
 D. 79 cm^2 E. 80 cm^2

29. L, M, and N are three distinct lines in a plane. Line L intersects line M. Lines M and N are parallel. How many points are the same distance from all three lines?

- A. 0 B. 1 C. 2 D. 4 E. More than 4

30. Triangle ABC has coordinates A(0, 0), B(8, 0), and C(0, 6). Triangle ABC is rotated 90° counter-clockwise about point B(8, 0), forming triangle $A'B'C'$ [point A has moved to A' ; B is renamed B' ; and C has moved to C']. Then triangle $A'B'C'$ is rotated 120° clockwise about point A' , forming triangle $A''B''C''$. As point C moved to C' and then to C'' , it traced two curved paths. What is the sum of the lengths of those two paths?

- A. $\frac{17}{3}\pi$ B. $\frac{29}{3}\pi$ C. $\frac{35}{3}\pi$ D. 7π E. 9π

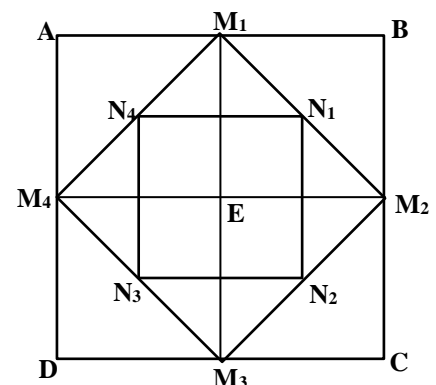
31. In the 3-person version of the game “**Rock, Paper, Scissors**”, assume that each person randomly chooses “Rock”, “Paper”, or “Scissors”, and then all three choices are announced [or “shown”] at the same time. With three people, the game ends after the first round only if:

- one person chooses *Rock* and the other two choose *Scissors*
- OR one person chooses *Scissors* and the other two choose *Paper*
- OR one person chooses *Paper* and the other two choose *Rock*.

Calculate the probability that the game does end after the first round.

- A. $8/27$ B. $1/3$ C. $11/27$ D. $1/9$ E. $2/9$

32. In the diagram, ABCD is a square with center E. All points labeled M_i or N_i are midpoints of their respective edges.



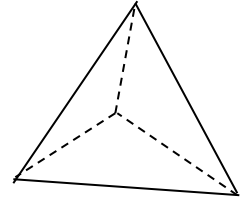
- How many ways are there to choose a sequence of three squares so that the:
- The interior of the smallest square you choose is inside the medium square you choose
 - and the interior of the medium square you choose is inside the largest square you choose?

- A. 8 B. 9 C. 12 D. 13 E. 17

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33. If x , y , and z are integers and $(x + 7)^2 + (y + 1)^2 + (z - 7)^2 = 133$, what is the least possible value of the sum $x + y + z$?
- A. -20 B. -17 C. -12 D. 4 E. 12

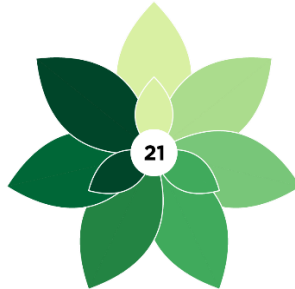
34. On each of the four faces of a regular tetrahedron, the midpoints of two different edges of the face are randomly selected and connected with a line segment. What is the probability that three of those four segments form an equilateral triangle?
- A. $1/27$ B. $2/27$ C. $1/9$ D. $4/27$ E. $2/9$



35. On a poster, Nick Rougeux has created a beautiful way to depict the prime factorization of numbers from 2 through 101. His depictions for 7, 21, and 24 are shown.



7



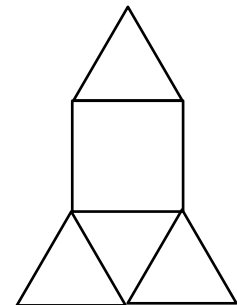
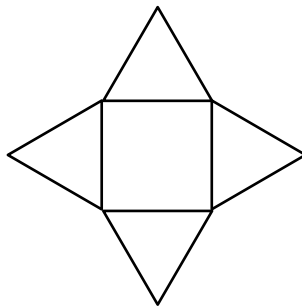
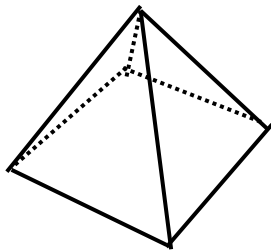
3 x 7



2 x 2 x 2 x 3

Following the same pattern, how many of Nick's 100 flowers will have exactly 12 petals?

- A. 3 B. 4 C. 5 D. 6 E. More than 6
- 36.



This square-based pyramid has four equilateral triangles as faces. Two possible "nets" for this pyramid are shown. A "net" is a set of one square and four equilateral triangles which could be folded to form this pyramid. As shown, any two pieces of a net must meet along an edge [not just a vertex].

Including these two nets, are many total nets are possible?

[Two nets are "different" if one net cannot be rotated and/or flipped onto the other net.]

- A. 6 B. 7 C. 8 D. 9 E. More than 9