St. Louis Community College
Meramec

MATH REVIEW SUPPLEMENT

for The

INTERMEDIATE ALGEBRA SECTION

of the

ACCUPLACER Entry Assessment

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College Level Math Study Guide for the Accuplacer(CPT)

The following sample questions are similar to the format and content of questions on the Accuplacer College Level Math test. Reviewing these samples will give you a good idea of how the test works and just what mathematical topics you may wish to review before taking the test itself. Our purposes in providing you with this information are to aid your memory and to help you do your best.

I. Factoring and expanding polynomials

Factor the following polynomials:

1. $15a^3b^2 - 45a^2b^3 - 60a^2b$
2. $7x^3y^3 + 21x^2y^2 - 10x^3y^2 - 30x^2y$
3. $6x^4y^4 - 6x^3y^2 + 8xy^2 - 8$
4. $2x^2 - 7xy + 6y^2$
5. $y^4 + y^2 - 6$
6. $7x^3 + 56y^3$
7. $81r^4 - 16s^4$
8. $(x+y)^2 + 2(x+y) + 1$

Expand the following:

9. $(x+1)(x-1)(x-3)$
10. $(2x+3y)^2$
11. $(\sqrt{3}x + \sqrt{3})(\sqrt{6}x - \sqrt{6})$
12. $(x^2 - 2x + 3)^2$
13. $(x+1)^5$
14. $(x-1)^6$

II. Simplification of Rational Algebraic Expressions

Simplify the following. Assume all variables are larger than zero.

1. $3^2 + 5 - \sqrt{4} + 4^0$
2. $9 \div 3.5 - 8 \div 2 + 27$
3. $\sqrt{\dfrac{81}{x^4}}$
4. $2\sqrt{18} - 5\sqrt{32} + 7\sqrt{162}$
5. $\dfrac{6x-18}{3x^2+2x-8} \cdot \dfrac{12x-16}{4x-12}$

III. Solving Equations

A. Linear

1. $3 - 2(x-1) = x - 10$
2. $\dfrac{x}{2} - \dfrac{x}{7} = 1$
3. $y(y+2) = y^2 - 6$
4. $2[x - (1-3x)] = 3(x+1)$

B. Quadratic & Polynomial

1. $\left(y - \dfrac{8}{3}\right)\left(y + \dfrac{2}{3}\right) = 0$
2. $2x^3 - 4x^2 - 30x = 0$
3. $27x^3 = 1$
4. $(x-3)(x+6) = 9x + 22$
5. $t^2 + t + 1 = 0$
6. $3x^3 = 24$
7. $(x+1)^2 + x^2 = 25$
8. $5y^2 - y = 1$

C. Rational

1. $\frac{1}{y-1}+\frac{2}{y+1}=0$
2. $\frac{2}{x-3}-\frac{3}{x+3}=\frac{12}{x^2-9}$
3. $\frac{1}{6-x}+\frac{2}{x+3}=\frac{5x}{x^2-3x-18}$
4. $\frac{11}{x^2-25}-\frac{2}{x-5}=\frac{1}{x+5}$
5. $\frac{1}{a}=\frac{-6}{a^2+5}$
6. $\frac{-1}{x^2-3x}=\frac{1}{x}+\frac{x}{x-3}$

D. Absolute Value

1. $|5-2z|-1=8$
2. $|x+5|-7=-2$
3. $|5x-1|=-2$
4. $\left|\frac{1}{2}x-\frac{3}{4}\right|=\frac{1}{4}$
5. $|y-1|=|7+y|$

E. Radicals

1. $4\sqrt{2y-1}-2=0$
2. $\sqrt{2x+1}+5=8$
3. $\sqrt{5x-1}-2\sqrt{x+1}$
4. $\sqrt{x^2+9}+x+1=0$
5. $\sqrt[3]{3x+2}+4=6$
6. $\sqrt[4]{w^2+7}=2$

IV. Solving Inequalities

Solve the following inequalities and express the answer graphically and using internal notation.

A. Linear

1. $\frac{3}{5}x+4\le -2$
2. $3(x+3)\ge 5(x-1)$
3. $3(x+2)-6>-2(x-3)+14$
4. $2\le 3x-10\le 5$

B. Absolute Value: Solve and Graph.

1. $|4x+1|\le 6$
2. $|4x+3|+2>9$
3. $\left|\frac{x+5}{x}\right|\ge 5$
4. $|5-2x|<15$

V. Lines & Regions

1. Find the x and y-intercepts, the slope, and graph $6x+5y=30$.
2. Find the x and y-intercepts, the slope, and graph $x=3$.
3. Find the x and y-intercepts, the slope, and graph $y=-4$.
4. write in slope-intercept form the line that passes through the points (4,6) and (-4,2).
5. write in slope-intercept form the line perpendicular to the graph of $4x-y=-1$ and containing the point(2,3).
6. Graph the solution set of $x-y\ge 2$.
7. Graph the solution set of $-x+3y<-6$.

VI. Graphing Relations, Domain & Range

For each relation, state if it is a function, state the domain & range.

1. $y = \sqrt{x+2}$
2. $y = \sqrt{x} - 2$
3. $y = \dfrac{x-1}{x+2}$
4. $f(x) = -|x+1| + 3$
5. $f(x) = \dfrac{2x-5}{x^2-9}$
6. $x = y^2 + 2$
7. $y = x^2 + 8x - 6$
8. $y = \sqrt{-x}$
9. $y = 3^x$
10. $h(x) = \dfrac{6x^2}{3x^2 - 2x - 1}$

VII. Exponents and Radicals

Simplify. Assume all variables are>0. Rationalize the denominators when needed.

1. $\sqrt[3]{-8x^3}$
2. $5\sqrt{147} - 4\sqrt{48}$
3. $\sqrt{5}\left(\sqrt{15} - \sqrt{3}\right)$
4. $\left(\dfrac{x^{2/3}y^{-4/3}}{x^{-5/3}}\right)^3$
5. $\sqrt[3]{\dfrac{40x^4}{y^9}}$
6. $\left(\dfrac{54a^{-6}b^2}{9a^{-3}b^8}\right)^{-2}$
7. $\dfrac{\sqrt[3]{27a^3}}{\sqrt[3]{2a^2b^2}}$
8. $\dfrac{2}{\sqrt{5} - \sqrt{3}}$
9. $\dfrac{x}{\sqrt{x} + 3}$

VIII. Complex Numbers

perform the indicated operation and simplify.

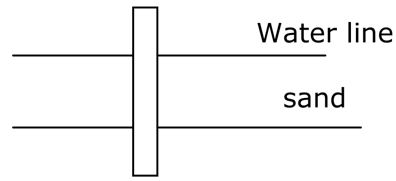
1. $\sqrt{-16} - 4\sqrt{-9}$
2. $\sqrt{-16}.\sqrt{-9}$
3. $\dfrac{\sqrt{-16}}{\sqrt{-9}}$
4. $(4-3i)(4+3i)$
5. $(4-3i)^2$
6. i^{35}
7. $\dfrac{3-2i}{4+5i}$

IX. Systems of Equations & Matrices

1. solve the systems: $\begin{aligned} 2x + 3y &= 7 \\ 6x - y &= 1 \end{aligned}$

X. Story problems

1. Sam made $10 more than twice what pete earned in one month. If together they earned $760, how much did each earn that month?
2. A women burns up three times as many calories running as she does when walking the same distance. If she runs 2 miles and walks 5 miles to burn up a total of 770 calories, how many calories does she burn up while running 1 mile?
3. A pole is standing in a small lake. If one-sixth of the length of the pole is in the sand at the bottom of the lake, 25 ft are in the water, and two-thirds of the total length is in the air above the water, what is the length of the pole?



XI. Functions

Let $f(x) = 2x + 9$ and $g(x) = 16 - x^2$. Find the following.

1. $f(-3) + g(2)$
2. $f(5) - g(4)$
3. $f(-1) \cdot g(-2)$
4. $\dfrac{f(5)}{g(5)}$

Answers

I. Factoring and Expanding polynominals

when factoring, there are three steps to keep in mind.

1. Always factor out the Greatest Common Factor
2. Factor what is left
3. If there are four terms, consider factoring by grouping.

Answers:

1. $15a^2b(ab - 3b^2 - 4)$
2. $7x^3y^3 + 21x^2y^2 - 10x^3y^2 - 30x^2y$ since there are 4 terms, we consider factoring by grouping. First, take out the Greatest Common Factor.

   $x^2y(7xy^2 + 21y \quad 10xy \quad 30)$

   $x^2y[(7xy^2 + 21y) + (-10xy - 30)]$ When you factor by grouping, be careful of the minus sign between the two middle terms.

   $x \quad y[7y(xy + 3) - 10(xy + 3)]$

   $x \quad y(7y - 10)(xy + 3)$

3. $2(3x^3y^2 + 4)(xy^2 - 1)$
4. $(2x - 3y)(x - 2y)$
5. $4 \quad -$

   When a problem looks slightly odd, we can make it appear more natural to us by using substitution (a procedure needed for calculus). Let $u = y^2$. Factor the expression with u's. Then, substitute the Y^2 back in place of the u's. If you can factor more, proceed. Otherwise, you are done.

   $u^2 + u - 6$

   $(u - 2)(u + 3)$

   $(y^2 - 2)(y^3 + 3)$

6. $7(x + 2y)(x^2 - 2xy + 4 \quad ^2$

   Formula for factoring the sum of two cubes:

   $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

   The difference of two cubes is:

   $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

7. $(3r - 2s)(3r + 2s)(9r^2 + 4s^2)$
8. $(x + y + 1)^2$ Hint: Let $u = x+y$
9. $x^3 - 3x^2 - x + 3$
10. $4x^2 + 12xy + 9y^2$
11. $3\sqrt{2}x^2 - 3\sqrt{2}$
12. $x^4 - 4x^3 + 10x^2 - 12x + 9$
13. $x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$
14. $x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1$

When doing problems 13 and 14, you may want to use pascal's Triangle.

II. Simplification of Rational Algebraic Expressions

1. 13
2. 38
3. $\frac{9}{x^2}$
4. $49\sqrt{2}$
5. $\frac{6}{x+2}$

If you have $\sqrt{4}$, you can write 4 as a product of primes(2.2). In square roots, it takes two of the same thing on the inside to get one thing on the outside: $\sqrt{4} = \sqrt{2.2} = 2$.

III. Solving Equations

A. Linear

1. $x = 5$ 2. $x = \frac{14}{5}$ or $2\frac{4}{5}$ 3. $y = -3$ 4. $x = 1$

B.Quadratic & Polynomials

1. $y = \frac{8}{3}, -\frac{2}{3}$
2. $x = 0, -3, 5$
3. $x = \frac{1}{3}, \frac{-1 \pm i\sqrt{3}}{6}$
4. $x = 10, -4$
5. $t = \frac{-1 \pm i\sqrt{3}}{2}$
6. $x = 2, -1 \pm i\sqrt{3}$
7. $x = 3, -4$
8. $y = \frac{1 \pm \sqrt{21}}{10}$

Solving quadratics or polynomials:

1. Try to factor
2. If factoring is not possible, use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ where } ax^2 + bx + c = 0.$$

Note: $i = \sqrt{-1}$ and that $\sqrt{-12} = i\sqrt{12} = i\sqrt{2.2.3} = 2i\sqrt{3}$

C. Rational

1.

$$\frac{1}{y-1}+\frac{\quad}{y+1}=0$$

$$(y-1)(y+1)\left[\frac{1}{y-1}+\frac{2}{\quad 1}\right] \quad 0(y-1)(y+1)$$

$$(y-1)(y+1)\frac{1}{y \quad 1}+(y-1)(y+1)\frac{2}{\quad} \quad 0$$

$$(y+1)+2(y-1)=0$$

$$3y-1=0$$

$$=\frac{1}{3}$$

Solving Rational Equations:
1. Find the lowest common denominator for all fractions in the equation
2. Multiply both sides of the equation by the lowest common denominator
3. Simplify and solve for the given variable
4. Check answers to make sure that they do not cause zero to occur in the denominators of the original equation.

2. Working the problem, we get x=3. However, 3 causes the denominators to be zero in the original equation. Hence, this problem has no solution.
3. $x=-\frac{15}{4}$
4. $x=2$
5. $a=-1,-5$
6. $x=-2,1$

D. Absolute Value

1.

$$| \quad z|$$

$$|5-2z|=9$$

$$5-2z= \quad \text{or } 5-2z= \quad 9$$

$$-2z=4 \text{ or } -2z=-14$$

$$z=-2 \text{ or } z=7$$

Solving Absolute Value Equations:
1. Islolate the absolute value on one side of the equation and everything else on the other side.
2. Remember that $|x|=2$ means that the object inside the absolute value has a distance of 2 away from zero. The only numbers with a distance of 2 away from zero are 2 and −2. Hence, $x=2$ or $x=-2$. Use the same thought process for solving other absolute value equations.

2. $x=0$ or $x=-10$
3. No solution! An absolute value can not equal a negative number.
4. $x=2$ or 1
5.

$$| \quad -1| \quad |7 \quad |$$

$$1= \quad + \quad \text{or y} \quad 1=- \ 7+\text{y}$$

$$0=8 \qquad \text{or } 2\text{ y}=-6$$

$$\text{solution} \quad \text{or} \quad y \quad -3$$

Hence, y = -3 is the only solution.

Note: An absolute value can not equal a negative value. $|x|=-2$ does not make any sense.

Note: Always check your answers!!

E. Radicals

1.

Solving Equations with radicals:
1. Isolate the radicals on one side of the equation and everything else on the other side.
2. If it is a square root, then square both sides. If it is a cube root, then cube both sides, etc….
3. Solve for the given variable and check your answer.

Note: A radical with an even index such as $\sqrt{\ }, \sqrt[4]{\ }, \sqrt[6]{\ }$, ……can not have a negative argument (The square root can but you must use complex numbers).

2. $x = 4$

3.

$$\sqrt{5x-1} - 2\sqrt{x+1} = 0$$
$$\sqrt{5x-1} = 2\sqrt{x+1}$$
$$\left(\sqrt{5x-1}\right)^2 = \left(2\sqrt{x+1}\right)^2$$
$$5x-1 = 4(x+1)$$
$$x = 5$$

Checking your answer you find that $4i - 2i\sqrt{2} = 0$. Since this is not possible, there is no solution for this problem.

4. No solution $x = 4$ does not work in the original equation.

5. $x = 2$

6. $w = 3, -3$

IV. Solving Inequalities

A. Linear

1.

When solving linear inequalities, you use the same steps as solving an equation. The difference is when you multiply or divide both sides by a negative number, you must change the direction of the inequality.
For example:

$$5 > 3$$
$$-1(5) < -1(3)$$
$$-5 < -3$$

Internal Notation: $(-\infty, -10]$

2. $x \le 7$ Internal Notation: $(-\infty, 7]$

3. $x > 4$ Internal Notation: $(4, \infty)$

4. $4 \le x \le 5$ Internal Notation: $[4,5]$

B. Absolute Value

1.

Think of the inequality sign as an alligator. If the alligator is facing away from the absolute value sign such as, $|X| < 5,$ then one can remove the absolute value and write $-5 < X < 5.$ This expression indicates that *X* can not be farther than 5 units away from zero.

If the alligator faces the absolute value such as, $|X| > 5,$ then one can remove the absolute value and write $X > 5$ or $X < -5$. These expressions express that *X* can not be less than 5 units away from zero.

Internal Notation: $[-\frac{7}{4}, \frac{5}{4}]$

2.

$x > 1 \text{ or } x < -\frac{5}{2}$

$\text{Interval}: \left(-\infty, -\frac{5}{2}\right) \cup (1, \infty)$

3.

$x \quad -20 \text{ or } x$

$\text{Interval}: (-\infty, -20] \cup [10, \infty)$

4.

$-5 < x < 10$

$\text{Interval}: (-5, 10)$

V. Lines and Regions

1. *x*-intercept: (5,0)
y-intercept: (0,6)

slope: $-\frac{6}{5}$

2. *x*-intercept: (3,0)
y-intercept: None
slope: None

3. *x*-intercept: None
y-intercept: (0,-4)
slope:0

4. $y = \frac{1}{2}x + 4$

5. $y = -\frac{1}{4}x + 3\frac{1}{2}$

6. $x - y \geq 2$

7. $-x + 3y < -6$

VI. Graphing Relations

1.

$$y = \sqrt{x+2}$$
Domain : $[-2, \infty)$
Range : $[0, \infty)$

2.

$$y = \sqrt{x} - 2$$
Domain : $[0, \infty)$
Range : $[-2, \infty)$

3.

$$y = \frac{x-1}{x+2}$$
Domain : All real Numbers except - 2
Range : $(-\infty, 1) \cup (1, \infty)$

4.

$$f(x) = -|x+1| + 3$$
Domain : $(-\infty, \infty)$
Range : $(-\infty, 3]$

5.

$$f(x) = \frac{2x-5}{x^2-9}$$
Domain : All Real Numbers except ± 3
Range : All Real Numbers.

6.

$$x = y^2 + 2$$
Domain : $[2, \infty)$
Range : $(-\infty, \infty)$

7. $= x^2 + 8x - 6$

$\text{Domain}: (-\infty, \infty)$

$\text{Range}: [-6, \infty)$

8.

$y = \sqrt{-x}$

$\text{Domain}: (-\infty, 0]$

$\text{Range}: [0, \infty)$

9.

$y = 3^x$

$\text{Domain}: (-\infty, \infty)$

$\text{Range}: (0, \infty)$

10.

$$h(x) = \frac{6x^2}{3x^2 - 2x - 1}$$

$\text{Domain: All Real Numbers except } -\frac{1}{3}, 1$

$\text{Range}: (2, \infty) \cup (-\infty, 0]$

VII. Exponents and Radicals

1. $-2x$
2. $5\sqrt{147} - 4\sqrt{48} = 35\sqrt{3} - 16\sqrt{3} = 19\sqrt{3}$
3. $5\sqrt{3} - \sqrt{15}$
4. $\frac{x^7}{y^4}$
5. $\frac{2x\sqrt{5x}}{y^3}$
6. $\left(\frac{54a^{-6}b^2}{9a^{-3}b^8}\right)^{-2} = \left(\frac{6}{a^3b^6}\right)^{-2} = \frac{a^6b^{12}}{36}$

7. $\frac{\sqrt[3]{27a^3}}{\sqrt[3]{2a^2b^2}} = \frac{3a}{\sqrt[3]{2a^2b^2}} = \frac{3a}{\sqrt[3]{2a^2b^2}} \cdot \frac{\sqrt[3]{4ab}}{\sqrt[3]{4ab}} = \frac{3a\sqrt[3]{4ab}}{2ab} = \frac{3\sqrt[3]{4ab}}{2b}$

8. $\sqrt{5} + \sqrt{3}$

9. $\left(\frac{x}{\sqrt{x}+3}\right)\left(\frac{\sqrt{x}-3}{\sqrt{x}-3}\right) = \frac{x\sqrt{x}-3x}{x-9}$

VIII. Complex Numbers

1. $\sqrt{-16} - 4\sqrt{-9} = 4i - 12i = -8i$

2. $\sqrt{-16}.\sqrt{-9} = (4i)(3i) = 12i^2 = -12$

3. $\frac{\sqrt{-16}}{\sqrt{-9}} = \frac{4i}{3i} = \frac{4i}{3i}.\frac{3i}{3i} = \frac{12i^2}{9i^2} = \frac{-12}{-9} = \frac{4}{3}$

4. $(4-3i)(4+3i) = 16 - 9i^2 = 16 + 9 = 25$

5. $(4-3i)^2 = (4-3i)(4-3i) = 16 - 24i + 9i^2 = 16 - 24i - 9 = 7 - 24i$

6. $i^{25} = i.i^{24} = i(i^2)^{12} = i(-1)^{12} = i$

7. $\frac{3-2i}{4+5i}.\frac{4-5i}{4-5i} = \frac{12-19i+10i^2}{16-25i^2} = \frac{12-19i-10}{16+25} = \frac{2-19i}{41}$

IX. Systems of Equations

1. $x = \frac{1}{2}$

$y = 2$

X. Story problem

1. $(2x+10) + x = 760$

2.

Answer: 210 Calories

3. Let x= length of pole $\frac{2}{5}x + 25 + \frac{1}{6}x = x$ Answer: 150 feet

XI. Functions

1. $f(-3)+g(2)=3+12=15$
2. $f(5)-g(4)=19-0=19$
3. $f(-1).g(-2)=7.12=84$
4. $\dfrac{f(5)}{g(5)}=-\dfrac{19}{9}$