Sixth Grade Test - Excellence in Mathematics Contest – 2014

1. Using one of the tables in Ogilvie’s *Ready Reckoner*, published in 1916, a worker earning 22½ cents per hour would earn $13.50 in one week. How many hours of work does this represent?
   A. 44  B. 48  C. 52  D. 56  E. 60

2. This diagram is the logo of an organization that is trying to stop the killing of elephants for their ivory tusks.

Each small 1 by 1 square represents one elephant per day killed in Africa in 2013.

How many 1 by 1 squares are in the diagram?
   A. 80  B. 96  C. 100
   D. 120  E. 125

3. When you multiply 625 million by 48 trillion, the product ends in how many zeroes?
   A. 15  B. 18  C. 19  D. 21  E. 22

4. On February 8, 2014, American Sage Kotsenburg won the first ever Olympic gold medal in a snow board event called *Slope Style*. In his trick named the Holy Grail, he rotated 1620 degrees.
   How many rotations is 1620 degrees?
   A. 3  B. 3.5  C. 4  D. 4.5  E. 9

Problems #5, #6, and #7 are from Buehrle’s 1886 *Exercises in Arithmetic*.

5. A farmer traded 267 bushels of peaches worth $1.75 per bushel for coal worth $5.30 per ton. Rounded to the nearest ton, how many tons of coal did he receive?
   A. 88  B. 144  C. 554  D. 809  E. 2476

6. Harry has 8 hens; each lays an average of 120 eggs a year. The barley on which he feeds them costs him $8.75 per year. He sells his eggs at 20 cents a dozen. In one year, what profit does Harry make from his hens?
   A. $7.25  B. $16  C. $24.75  D. $114.50  E. $183.25

7. One acre of land produces 30,000 pounds of beets. What is the value of the beet crop from 3 acres of land if the beets can be sold at $2.00 per one thousand pounds?
   A. $45  B. $180  C. $20,000  D. $45,000  E. $180,000

8. A rectangle is twice as long as it is wide. Its perimeter is 72 cm. What is its area in square centimeters?
   A. 96  B. 144  C. 256  D. 288  E. 324
Problems #9, #10, and #11, below appeared in the category Math Problems on a round of Double Jeopardy on the January 30, 2014, episode of Jeopardy. Amazingly, no contestant answered any of these three questions correctly. Can you?

9. A person was in prison for every day of the years 2009 to 2013, inclusive. For how many first days of a month was this person in prison?
A. 36  B. 47  C. 48  D. 49  E. 60

10. After Barry spends 20% of his savings, he has $200. How much money did Barry start with?
A. $240  B. $250  C. $280  D. $1000  E. $1200

11. How much does it cost to carpet a rectangular 10 foot by 18 foot room if carpet costs $10 per square yard?
A. $180  B. $200  C. $560  D. $600  E. $1800

12. This is the 36th Annual Excellence in Mathematics contest. From the cover of this test, note that 36 is the 6th square number and the 8th triangular number. What is the sum of the 10th square number and the 10th triangular number?
A. 110  B. 145  C. 150  D. 155  E. 200

13. Two buses travel the same route. The first bus goes 50 miles per hour and leaves at 2:00 PM. The second bus goes 40 miles per hour and leaves the same station at 3:30 PM. How many miles apart are they at 5:15 PM?
A. 2.5  B. 55  C. 67.5  D. 80  E. 92.5

14. If \[57 + 67 + 77 + 87 + 97 + 107 = 27 + 37 + 47 + 57 + 67 + 77 + N,\] then what does N equal?
A. 90  B. 120  C. 150  D. 180  E. 210

15. \[-20; 4; 7; -9; ...

Each term of this sequence is the sum of the previous three terms. For example, \[-20 + 4 + 7 = -9.\] The first four terms are given above. What is the tenth term?
A. -61  B. -24  C. 0  D. 4  E. 12

16. How many \[\frac{1}{16}\]s are in \[2 \frac{3}{4}\]
A. 36  B. 38  C. 40  D. 42  E. 44

17. At 3:30 AM, what is the measure in degrees of the angle between the minute hand and the hour hand on a standard 12-hour clock?
A. 90°  B. 60°  C. 67.5°  D. 75°  E. 82.5°
18. Which one of these four numbers is the largest?

A. \( \sqrt{\frac{1}{49}} \)  
B. \( \left( \frac{1}{49} \right)^2 \)  
C. \( \frac{1}{\sqrt{49}} \)  
D. \( \frac{1}{\left( \frac{1}{49} \right)^2} \)

E. None, because two of these four numbers are equal and are tied for the largest number

19. Peter weighs 112 pounds. For his 12th birthday, he asks for the number of nickels that would weigh 112 pounds. If his parents agree, what is the value of the money he will receive?

A nickel weighs 0.176 ounces. Round to the nearest dollar.

A. $16  
B. $128  
C. $420  
D. $509  
E. $10,182

20. For each rectangle ABCD and EFGH, its length is three times its width. If AB = 60 cm and FP = 12 cm, what is the total area of the shaded region?

A. 432 cm²  
B. 436 cm²  
C. 440 cm²  
D. 444 cm²  
E. 448 cm²

21. S, T, and L are natural numbers (not necessarily distinct) such that \( S \cdot T \cdot L = 60 \). What is the minimum possible value of the sum \( S + T + L \)?

A. 12  
B. 13  
C. 14  
D. 15  
E. 16

22. Of these four operation symbols, insert three different symbols (and no other symbols or numbers) into the three rectangles to make an equation. Be sure to follow the correct order of operations rules of arithmetic.

\[
\begin{array}{cccc}
\times & \div & - & + \\
12 & & 2 & 9 & 6 = 60
\end{array}
\]

Which symbol is NOT used?

A. \( \times \)  
B. \( \div \)  
C. \( - \)  
D. \( + \)  
E. An equation is not possible

23. According to Oxfam, in January 2014 the total wealth of the richest 85 people on Earth was 1.7 trillion dollars, the same amount as the total wealth of the poorest 3.5 billion people on Earth. Approximately what is the average wealth per person of those 3.5 billion people?

A. $6  
B. $500  
C. $5000  
D. $5700  
E. $6000

24. A prime number \( P \) is a Sophie Germaine prime if \( 2P+1 \) is also a prime number. How many of the first 10 prime numbers are Sophie Germaine primes?

A. 4  
B. 5  
C. 6  
D. 7  
E. 8
25. Here is one more problem from *Buehrle’s 1886 Exercises in Arithmetic*.

Five boys earn some money selling fish. The first boy earns 36 cents which is 1/5 of the whole amount earned; the second boy earns 1/6 of the whole; the third boy earns 1/4 of the whole; and the fourth boy earns the same as the second boy. If the fifth boy earns the remainder, how much does he receive?

A. 18¢  B. 24¢  C. 36¢  D. 39¢  E. 69¢

26. Ogilvie’s *Ready Reckoner*, published in 1916, provided the following rule for estimating the weight of a cow.

*In feet, measure the girth and the length of the cow. Multiply the square of the girth by five times its length. Its weight will be given in pounds by taking away one-third of this product.*

Using this rule, what is the weight in pounds of a cow with girth 8 feet and length 6 feet?

A. 57  B. 640  C. 1280  D. 1440  E. 1920

27. Given a circle C of radius r. What is the ratio of the area of a semi-circle of C to the circumference of circle C?

A. r  B. 2r  C. 4r  D. r/4  E. r/2

28. A large bucket contains eight quarts of Orange Drink which consists of 40% orange juice and 60% water. After two quarts of the Orange Drink are consumed, the host adds six quarts of pure orange juice. What is the per cent orange juice now in the bucket?

A. 43%  B. 66%  C. 67%  D. 70%  E. 75%

29. Note that \(1+3+5 = 3^2\) and that \(1+3+5+7 = 4^2\).

If \(1+3+5+7+9+\ldots+2005+2007+2009+2011+2013 = N^2\). What is N?

A. 1005  B. 1006  C. 1007  D. 1008  E. N is not a Natural Number

30. For how many five-digit natural numbers N is \(\sqrt{N}\) a natural number?

A. 208  B. 216  C. 217  D. 218  E. 316

31. A, B, and C are whole numbers such that \(\frac{A}{2} = \frac{B}{3} = \frac{C}{5}\) and \(A \cdot B \cdot C = 240\). What is the sum \(A+B+C\)?

A. 20  B. 21  C. 22  D. 23  E. 24
32. A **Collatz Sequence** starts with a natural number and is formed by these two rules:

- If a term \( N \) is even, the next term is \( N/2 \)
- If a term \( N \) is odd, the next term is \( 3N+1 \)

For example, \( 12, 6, 3, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, \ldots \) is a **Collatz Sequence**.

Write the **Collatz Sequence** that starts with \( 28 \). Stop when “1” first appears in your sequence. Including “28” and “1”, how many terms are in your sequence?

A. 11  B. 13  C. 15  D. 17  E. 19

33. In the 1980’s, statisticians analyzing data noticed that a trend that appeared in different groups of data could reveal an opposite trend when the data was combined into one group. They called this phenomenon **Simpson’s Paradox**. As a financial analyst at Google, Zan has studied the occurrence of **Simpson’s Paradox** in revenue data collected by Google. Here is an example of **Simpson’s Paradox** from baseball.

For a hitter in baseball, let \( H = \) number of hits and \( B = \) number of at-bats. Then a player’s **Batting Average** \( A \) is given by: \( A = H/B \) and this number is usually rounded to **three decimal places**.

Complete this table.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hits ( H )</td>
<td>At-Bats ( B )</td>
<td>Average ( A )</td>
</tr>
<tr>
<td>David Justice</td>
<td>104</td>
<td>411</td>
<td>0.253</td>
</tr>
<tr>
<td>Derek Jeter</td>
<td>12</td>
<td>48</td>
<td>0.250</td>
</tr>
</tbody>
</table>

Note that David Justice had a higher batting average in both 1995 and 1996. Compute \( X - Y \).

A. –0.040  B. –0.005  C. 0  D. 0.005  E. 0.040

34. Follow this set of three instructions:
   - If \( -5 \) is not a non-negative number, write 6 on your paper. Otherwise, write 9.
   - If March is a month or if elephants are red, write 7 on your paper. Otherwise, write \( -2 \).
   - If \( -64 \) is the cube of an integer, write \( -5 \). Otherwise, write 8.

What is the sum of the three numbers that you have written?

A. 2  B. 8  C. 11  D. 21  E. 24

35. Lillian has a large collection of 3 cm by 3 cm tiles and 2 cm by 2 cm tiles. She will use these tiles to completely cover a 7 cm by 18 cm rectangle. The tiles cannot overlap and the tiles cannot extend beyond the tray. She cannot break any of the tiles. How many tiles will she use?

A. 14  B. 24  C. 25  D. 28  E. It is impossible for Lillian to complete this task.
36. The Klog cereal company reduces the price of one box of Oat Flakes by 25%. However, they also reduce the number of ounces of cereal in the box by 10%. By what percent has the price per ounce of Oat Flakes been reduced?

A. 15%  
B. 16 2/3%  
C. 17.5%  
D. 20%  
E. 22.5%

37. Valerie and Ian have the following conversation.
Valerie, “If you gave me $1, we would then have the same amount of money.”
Ian replies, “If instead you gave me $6, I would have twice the amount of money that you would have.”

What is the total amount of money that Valerie and Ian have?

A. $38  
B. $42  
C. $46  
D. $50  
E. $54

38. Square ABCD with side length 6 cm is inscribed in a semi-circle.
What percent of the area of the semi-circle is covered by the square? Round to the nearest whole percent.

A. 48%  
B. 49%  
C. 50%  
D. 51%  
E. 52%

39. In a Motzkin path on a grid, three types of moves are allowed:
1. One unit right: →
2. One diagonal up: ↑
3. One diagonal down: ↓

One Motzkin path from A to B is shown on this 4 by 4 grid. Including the path shown in the diagram, how many different Motzkin Paths are there from A to B? (Note: All paths must stay on the grid.)

A. 7  
B. 8  
C. 9  
D. 10  
E. 11

40. See next page for problem #40.
Problem #40 below requires you to solve a Kakuro puzzle. Here is a Kakuro puzzle and its solution.

**RULES** for completing a Kakuro puzzle:

- Fill in each blank square with a natural number from 1 through 9.
- The sum of the numbers in consecutive white squares in each row or column is given.
- In each horizontal or vertical sum, a number CANNOT be repeated. For example, if two numbers sum to 16, they must be 7 and 9, not 8 and 8. (However, the same number could appear in the two sums in one row or column as in the 4th column in the above solution.)

**HINT:** Certain sums can be achieved in only one or two ways.
For example, if four numbers sum to 28, they must either be 4, 7, 8, and 9 or 5, 6, 8, and 9.

#40. Complete this Kakuro puzzle.

What is the **product** of the numbers in the squares labeled A, B, and C?

A. 320  
B. 336  
C. 384  
D. 432  
E. 504  

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